International Research Journal of Innovations in Engineering and Technology (IRJIET)

ISSN (online): 2581-3048

Volume 6, Issue 9, pp 82-87, September-2022

https://doi.org/10.47001/IRJIET/2022.609013

How to Make the Equation of the Hyperboloidal Helicoidal Surface and Some Applications of It

Manh Hong Do

Hanoi University of Science and Technology, No. 1, Dai Co Viet street, Hai Ba Trung distric, Hanoi, Viet Nam
School of Mechanic, Descritive Geometry and Engineering Drawing Department

Abstract **- Curves with equations (EQT) are widely used to create surfaces of machine parts in machine manufacture. They make beautiful surfaces that appeal to consumers. Helix and spiral curves are very popular applications in mechanical engineering and architecture. They not only are the basis for creating threaded surfaces on machine parts but also create beautiful spiral staircases. Based on the results of the 2019 paper, this paper shows how to make the equation of the hyperboloidal helicoidal surface and some projections of hyperboloit helix.**

Keywords: Equations, EQT Projection, PRJ Point, Pt.

I. INTRODUCTION

Curved surfaces and curves are widely used in engineering. They create very beautiful images on the surface of machine parts and architectural works. Among them is the one-sided hyperboloit. Humans have applied a single-sided hyperboloit to create lots of beautiful towers and structures. Spiral lines on curved surfaces are also very common in engineering. Cylindrical and conical helixes for creating thread surfaces on machine parts. The helix spiral is also used to create beautiful spiral stairs. The hyperboloithelix is applied to create soft curves in amusement parks. The current graphics software can only draw cylindrical helix and conical helix. This paper shows how to make the parameter equation of the hyperboloidal helicoidal surface.

II. MAKE EQUATION OF THE HYPERBOLOIT HELIX

2.1 A geometry Math

Given a point M moving at constant speed on straight line l

The line l rotates around the line t with a constant angular velocity

The result will produce 3 following cases

Case 1:

If line t is parallel to line l, the trajectory of Pt M will be a cylindrical spiral

Case 2:

If line t intersects line l, then the trajectory of Pt M will be a helical cone

Case 3:

If both lines t and l are skew lines, the Trajectory of the point M will be a Hyperboloit helix.

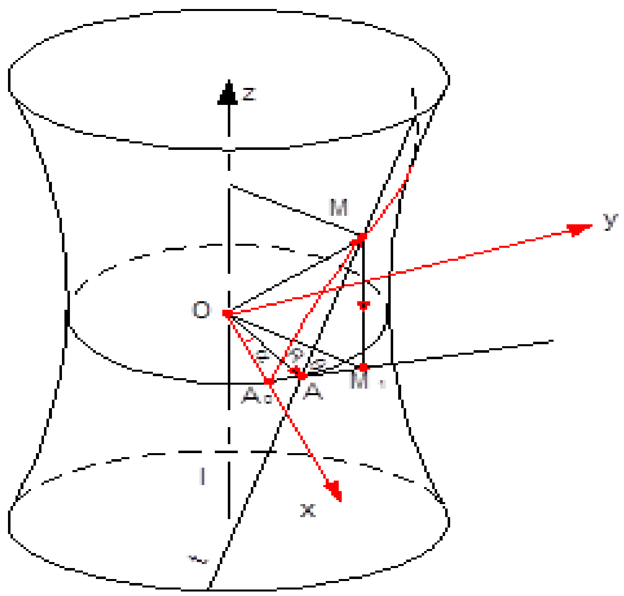

Figure 1

We have: OA=R

Line OA not only is a radius of the smallest circle but also perpendicular to both line t and line l.

α is rotation angle.

When α =0 then the first position of Pt M is the position of Pt Ao. Make a coordinate system oxyz: Ox≡OAo, Oz≡l

θ is the angle between line t and plane xoy.

When α ≠0 then Pt M move in line t.

We have: AM=kα, k=const.

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$

© 2022-2017 IRJIET All Rights Reserved

ISSN (online): 2581-3048

Volume 6, Issue 9, pp 82-87, September-2022

https://doi.org/10.47001/IRJIET/2022.609013

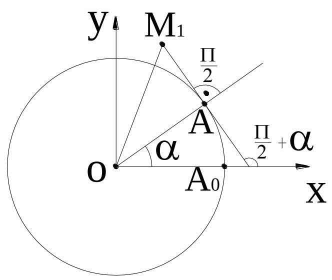

Figure 2

Projecting on the x-axis we have:

$$x = Rcos \propto + AMcos\theta \cos(\propto + \frac{\pi}{2})$$

$$\rightarrow x = Rcos \propto - AMcos\theta \sin \propto \ (1)$$

Projecting on the y-axis we have:

$$y = Rsin \propto + AMcos\theta \cos\left[\frac{\pi}{2} - (\propto + \frac{\pi}{2})\right]$$

$$\rightarrow y = Rsin \propto + AMcos\theta \cos \propto \quad (2)$$

Projecting on the z-axis we have:

$$z = 0 + AMcos\varphi$$

$$\rightarrow z = AMcos\varphi \quad (3)$$

φ is the angle between line t and z-axis, θ is the angle between line t and plane xoy. So, the value of them is constant. We put cosφ=b, cosθ=c and AM=kα then we has the EQT of helix-curve of hyperboloit of one sheet:

$$x = Rcos \propto - kc \propto \sin \propto \quad (4)$$

$$y = Rsin \propto + kc \propto \cos \propto \quad (5)$$

$$z = kb \propto \quad (6)$$

2.2 Some special cases

Case 1:

If the line l is parallel to the line t thenθ=90^0.

Replace this value into EQT (1), (2), (3):

$$x = Rcos \propto$$

$$x = Rcos \propto \quad (7)$$

$$y = Rsin \propto \quad (8)$$

$$z = b\alpha \quad (9)$$

This is an EQT of helix-sprail of a cylinder.

The EQT of the circle of its is:

$$x^2 + y^2 = R^2$$

Case 2

If line t intersects line l then OA=R=0.

Replace this value into EQT (1), (2), (3) :

$$x = -kc \propto \sin \propto (10)$$

$$y = kc \propto \cos \propto \quad (11)$$

$$z = b\alpha (12)$$

This is an EQT of helix-sprail of a cone.

The EQT of the circle of it is:

$$x^2 + y^2 = R^2 = k^2c^2\alpha^2 \quad (13)$$
$$R = kc \propto (14)$$

The property of a single layer hypecboloit surface is its tangent to a fixed direction at a constant angle. We will prove this property.

Derivative of the EQT (4), (5) and (6) we have:

$$\dot{x} = -Rsin \propto - kc\dot{\propto} cos \propto - kcsin \propto$$

$$\dot{x} = -(R + kc)sin \propto - kc \propto cos \propto (15)$$

$$\dot{y} = Rcos \propto + kc\dot{c}os \propto - kc \propto sin \propto \quad (16)$$

$$\dot{z} = kb (17)$$

III. PERPENDICULAR PROJECTIONS OF HYPERBOLOIT SPRIAL

3.1 Top view

The prj of a hyperboloit spiral on the XOY plane is:

$$x = Rcos \propto - kc \propto \sin \propto (18)$$

$$y = Rsin \propto + kc \propto \cos \propto$$

The x-axis is the symmetry axis of this curve

A value of x will give two values of y.

Fig 3 The top view of Hyperboloit spiral

© 2022-2017 IRJIET All Rights Reserved

International Research Journal of Innovations in Engineering and Technology (IRJIET)

ISSN (online): 2581-3048

Volume 6, Issue 9, pp 82-87, September-2022

https://doi.org/10.47001/IRJIET/2022.609013

3.2 Left view

The prj of a hyperboloit spiral on the XOY plane is:

$$y = R\sin\propto + kc\propto\cos\propto \qquad (19)$$

$$z = kb\propto$$

$$y = R\sin\alpha + kc\alpha\cos\alpha = R(\sin\alpha + \frac{kc\alpha}{R}\cos\alpha)$$

We set:

$$\frac{kc\alpha}{R} = tg\varphi = \frac{\sin\varphi}{\cos\varphi}$$

$$y = R(\sin\alpha + \frac{\sin\varphi}{\cos\varphi}\cos\alpha)$$

$$\rightarrow y = \mathrm{R}\left(\frac{\sin\alpha\cos\varphi + \cos\alpha\sin\varphi}{\cos\varphi}\right)$$

$$\rightarrow y = \mathrm{R}\frac{\sin(\alpha + \varphi)}{\cos\varphi}$$

$$\cos\varphi = \frac{1}{\sqrt{1 + tg^2\varphi}}$$

$$\cos\varphi = \frac{1}{\sqrt{1 + \frac{k^2c^2\alpha^2}{R^2}}}$$

$$\cos\varphi = \frac{R}{\sqrt{R^2 + k^2c^2\alpha^2}}$$

$$y = \sin(\alpha + \varphi)\sqrt{R^2 + k^2c^2\alpha^2} \qquad (20)$$

This is the EQT for the sine function. Its amplitude of oscillation is bounded by a hyperbol

We consider the function:

$$y^2 = R^2 + k^2C^2\propto^2$$

$$y = \pm\sqrt{R^2 + k^2c^2\propto^2} \qquad (21)$$

This is the function of the hyperbol

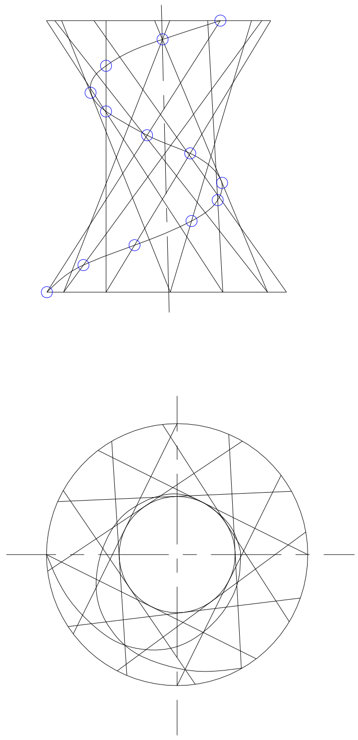

Figure 4: Two views of hyperboloid

Figure 4 shows how to draw 2 prj of the hyperboloit sprial. This drawing is based on the law of the formation of spiral. It is a combination of translational motion and rotational motion.

If we project on the plane that is parallel to the plane (XOY) in the direction of a generation, the curvinlinear curves of the hyperboloide surface will have a prj that is a beam of circles, passing through Pt A and B.

Pt A and Point B are the prj of two generations that parallel to the prj direction.

As for the sprial will have defferent prj and curve beam that also passing through Pt A and B. (See fig 5)

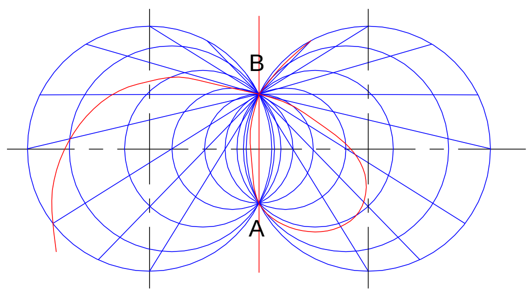

Figure 5

© 2022-2017 IRJIET All Rights Reserved

International Research Journal of Innovations in Engineering and Technology (IRJIET)

ISSN (online): 2581-3048

Volume 6, Issue 9, pp 82-87, September-2022

https://doi.org/10.47001/IRJIET/2022.609013

Those curves are prj of the sprials of the helix in figure 6.

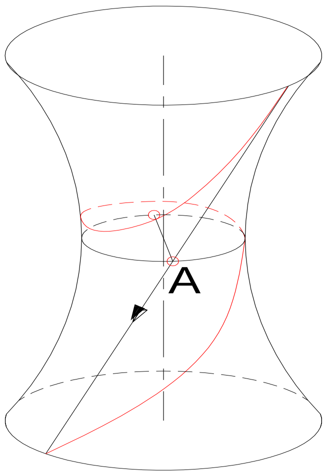

Figure 6

3.3 Front view

$$x = Rcos\alpha - kc\alpha sin\alpha = R(cos\alpha - \frac{kc\alpha}{R} sin\alpha)$$

$$\frac{kc\alpha}{R} = cotg\gamma = \frac{cos\gamma}{sin\gamma}$$

$$x = R(cos\alpha - \frac{cos\gamma}{sin\gamma} sin\alpha)$$

$$\rightarrow x = \mathrm{R}\left(\frac{sin\gamma cos\alpha - cos\gamma sin\alpha}{sin\gamma}\right)$$

$$\rightarrow x = \mathrm{R}\frac{sin(\gamma - \alpha)}{sin\gamma}$$

$$sin\gamma = \frac{1}{\sqrt{1 + cotg^2\gamma}}$$

$$sin\gamma = \frac{1}{\sqrt{1 + \frac{k^2c^2\alpha^2}{R^2}}}$$

$$sin\gamma = \frac{R}{\sqrt{R^2 + k^2c^2\alpha^2}}$$

$$x = \sin(\gamma - \alpha)\sqrt{R^2 + k^2c^2\alpha^2} \qquad (22)$$

This is the EQT for the sine function

Since its axis is perpendicular to the plane (XOY), so its front view and left side views are the same.

IV. HOW TO MAKE EQT OF THE HYPERBOLOIDAL HELICOINDAL SURFACE

All tangents to the hyperboloit sprial make a hyperboloidal helicoindal surface. We con set up the EQT of that surface as follows:

Suppose P is a Pt moving on the tangent PM. M is the tangent Pt. P has coordinates: X, Y, Z. (See fig 7)

We have:

$$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} \qquad (23)$$

$$\overrightarrow{OP} = \overrightarrow{OM} + v.\vec{t} \qquad (24)$$

$$\overrightarrow{OP} = \overrightarrow{OM} + \frac{v}{|T|}.|T|.\vec{t} \qquad (25) |\overrightarrow{T}| = \{\dot{x}(\alpha)\dot{y}(\alpha)\dot{z}(\alpha)\}$$

$$X = x(\alpha) + u\dot{x}(\alpha) \qquad (26)$$

$$Y = y(\alpha) + u\dot{y}(\alpha) \qquad (27)$$

$$Z = z(\alpha) + u\dot{z}(\alpha) \qquad (28)$$

$$\dot{x}(\alpha) = -R\ sin\alpha - kc\alpha\ cos\alpha + kcsin\alpha$$

$$\dot{x}(\alpha) = (kc - R)\ sin\alpha - kc\alpha\ cos\alpha \qquad (29)$$

$$\dot{y}(\alpha) = R\ cos\alpha - kc\alpha\ sin\alpha + k\ ccos\alpha$$

$$\dot{y}(\alpha) = (kc\alpha + R)cos\alpha - kc\alpha\ sin\alpha \qquad (30)$$

$$\dot{z}(\alpha) = kb \qquad (31)$$

So, we have:

$$X=Rcos \propto -kc \propto \sin \propto +u(kc-R)sin\alpha - ukc\propto cos\alpha \quad (32)$$

$$Y = Rsin \propto +kc \propto \cos \propto +u(kc\alpha + R)cos\alpha - ukc\alpha\ sin\alpha \quad (33)$$

$$Z=kb \propto+kb = kb(\alpha + 1) \qquad (34)$$

So, we have the parametric EQT of the hyperboloidal helicoidal surface:

$$X=(R - ukc \propto)cos \propto +(ukc - kc \propto -uR)sin\alpha \quad (35)$$

$$Y = (R - ukc \propto)sin \propto +(kc \propto +ukc \propto +uR)cos\alpha \quad (36)$$

$$Z= kb(\alpha + 1) \qquad (37)$$

© 2022-2017 IRJIET All Rights Reserved

ISSN (online): 2581-3048
Volume 6, Issue 9, pp 82-87, September-2022
https://doi.org/10.47001/IRJIET/2022.609013

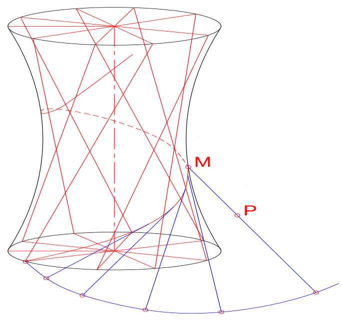

Figure 7

V. SOME APPLICATIONS OF HYPERBOLOIT OF ONE SHEET AND HYPERBOLOIT SPRIAL

Figure 8: The Killesberg observation tower, Stuttgart, Germany, 2001

Figure 9: Museum and event vanue, Munich, Germany 2007

Figure 10: Cathedral of Brazil, 1970

Figure 11: Essart-le –Roi, water tower of France
The Killesberg observation tower, Stuttgart, Germany, 2011
Hyperboloid single-sided hand

REFERENCES

[1] Nguyen Dinh Tri (editor-in-chief), Advanced Mathematics, volume 1, Education Publishing House 2006... Nguyen Thuy Thanh, Background of complex variable theory, VNU Publishing House, 2006.

[2] Dinh Tri Nguyen (editor) - Van Dinh Ta - Ho Quynh Nguyen ... Advanced Math Exercises Episode 1 - Dinh Tri Nguyen June 3, 2018In "Curriculum".

[3] Mechanics Program (Episode 2: Dynamics) – Sanh Do (Editor). September 2016 - publisher of Hanoi University of Science and Technology.

[4] https://en.wikipedia.org/wiki/Hyperboloid

AUTHOR'S BIOGRAPHY

Manh Hong Do M. E (1968), Degrees in mechanical engineering from Ha Noi University of Science and technology.
He is a Master, Department of Mechanical Engineering, Ha Noi University of Science and Technology.

© 2022-2017 IRJIET All Rights Reserved

ISSN (online): 2581-3048

Volume 6, Issue 9, pp 82-87, September-2022

https://doi.org/10.47001/IRJIET/2022.609013

Citation of this Article:

Manh Hong Do, "How to Make the Equation of the Hyperboloidal Helicoidal Surface and Some Applications of It" Published in *International Research Journal of Innovations in Engineering and Technology - IRJIET*, Volume 6, Issue 9, pp 82-87, September 2022. Article DOI https://doi.org/10.47001/IRJIET/2022.609013

****