

A Novel Method for Spatial Structural Kinematic Analysis

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Abstract - Kinematic analysis is the first task that needs to be solved for further real-life problems such as machine dynamics and durability, machine balancing, and optimization, chemistry, etc. Currently, there are many methods to analyze the kinematic problem, for instance: vector graphics method, vector-analytic method, or Denavit-Hartenberg-Craig matrix method. Although each method has its advantages, they have mostly encountered difficulties in establishing equations associated with complicated spatial structures. This research proposed a novel method, entitled "perpendicular projection method" to solve the problem of spatial structural kinematic analysis. This is an approximate solution method that combines the use of generalized coordinates and direction cosine matrices. With this method, the problem of kinematic analysis of spatial structures will be solved simply and easily by establishing connection equations as well as simulating the motion of spatial structures. This method was then applied in conducting the kinematic analysis of a spatial mechanism with 1 rotating joint (R), 1 ball joint (S), 1 translational joint (P), and one cylindrical joint (C). The calculated results, including the velocity, acceleration of generalized coordinates, angular velocity, angular acceleration of the links, velocity, and acceleration of the center of mass of the links, showed the efficiency of the proposed method.

Keywords: Spatial structure, Kinetic analysis, Perpendicular projection method, Generalized coordinates, Direction cosine matrices.

I. INTRODUCTION

The task of kinematics is to determine the position of the object at each time and the characteristics of motion. That is, it is necessary to build an expression describing the relationship between the positioning parameters (position-determining parameters) of the object concerning time, called the equation of motion. After that, it is necessary to determine the characteristics of motion, which are quantities that indicate the evolution of motion such as velocity, acceleration of point motion, angular velocity, and angular acceleration of motion of the object. Currently, there are many methods to analyze the kinematic problem, for instance: vector graphics method,

vector-analytic method, or Denavit-Hartenberg-Craig matrix method. The content of each method is described as follows.

1.1 Vector graphics method

a) Location problem:

Suppose we have a system of vector equations:

$$\begin{cases} \vec{m} = \vec{m}_1 + \vec{m}_2 + \dots + \vec{m}_n & (1) \\ \vec{m} = \vec{m}'_1 + \vec{m}'_2 + \dots + \vec{m}'_n & (2) \end{cases}$$

Vectors $\vec{m}, \vec{m}_1, \vec{m}'_1$ share the same root; vectors $\vec{m}, \vec{m}_n, \vec{m}'_n$ share the same tip. From there we see that if in equation (1) we completely know the vectors $\vec{m}_1, \vec{m}_2, \dots, \vec{m}_{n-1}$ and know the direction of the vector \vec{m}_n . In equation (2), knowing completely the vectors $\vec{m}'_1, \vec{m}'_2, \dots, \vec{m}'_{n-1}$ and knowing the direction of the vector, \vec{m}'_n we can use vector diagrams to determine the vector \vec{m} .

b) Velocity relationship:

As shown in Figure 1, in case two points A and B are on the same stitch, we have

$$\vec{v}_A = \vec{v}_B + \vec{V}_{BA} \quad (3)$$

Where:

\vec{v}_A, \vec{v}_B is the absolute velocity of points B and A

\vec{V}_{BA} is the relative velocity of B when rotating around point A

\vec{V}_{BA} perpendicular to BA, in the direction of rotation ω , value equal to $\omega \cdot AB$.

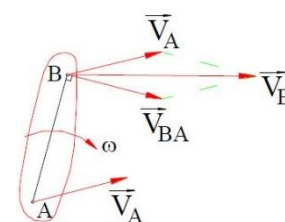


Figure 1: Velocity relationship in case two points A and B are on the same stitch

As shown in Figure 2, in case two points B_i and B_k coincide immediately on two stages i and k , we have:

$$\vec{v}_{B_i} = \vec{v}_{B_k} + \vec{v}_{B_i B_k}^r \quad (4)$$

Where:

$\vec{v}_{B_i}, \vec{v}_{B_k}$ is the absolute velocity of points on the two links

$\vec{v}_{B_i B_k}^r$ is the velocity in the relative motion of B_i with B_k , with a direction parallel to the translation direction between stage i and stage k .

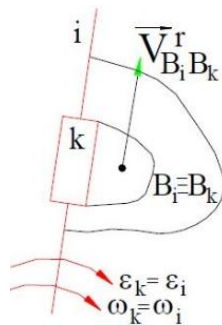


Figure 2 Velocity relationship in case two points B_i and B_k coincide immediately on two stages i and k

c) Acceleration relationship:

When two points A and B are on the same line (Figure 3)

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \vec{a}_A + \vec{a}_{BA}^n + \vec{a}_{BA}^t \quad (5)$$

Where:

\vec{a}_A, \vec{a}_B is the absolute acceleration of points A and B.

\vec{a}_{BA} is the acceleration in the relative motion of B around A

\vec{a}_{BA}^n is the normal acceleration component in the direction from B to A with value equal to $\omega^2 AB$.

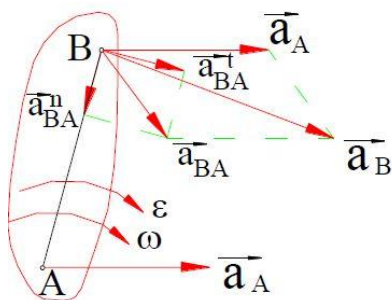


Figure 3: Acceleration relationship in case two points A and B are on the same line

When the two points B_i and B_k coincide instantly on the two links i and k (Figure 4)

$$\vec{a}_{B_i} = \vec{a}_{B_k} + \vec{a}_{B_i B_k}^k + \vec{a}_{B_i B_k}^t \quad (6)$$

Where:

$\vec{a}_{B_i}, \vec{a}_{B_k}$ is the absolute acceleration of points B_i and B_k .

$\vec{a}_{B_i B_k}^k = 2\vec{\omega}_i \times \vec{v}_{B_i B_k}^r$ is the Coriolis acceleration in the relative motion of B_k and B_i .

$\vec{a}_{B_i B_k}^t$ is the acceleration in the relative motion of B_i and B_k .

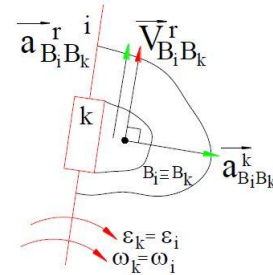


Figure 4: The two points B_i and B_k coincide instantly on the two links i and k

1.2 Vector - analytic method

This method is based on the vector equation of the structure. That vector equation is then projected onto the coordinate system to get the equation of motion.

Take the example of kinematic analysis of a 4-link structure as shown in Figure 5. The dynamic diagram of the 4-link planar all-articular low-joint structures has the form of a quadrilateral (in some cases, degenerates into a triangle). If we represent the edges of this dynamic diagram polygon with consecutive vectors, we will get a closed vector chain. Let l_i be the i th vector of the series, we have the following vector equation:

$$\sum_{i=1}^4 l_i = 0 \quad (7)$$

Let e_i, l_i be the unit vectors indicating the direction and length of vector l_i respectively, then equation (7) can be rewritten as follows:

$$\sum_{i=1}^4 l_i e_i = 0 \quad (8)$$

Equations (7), (8) are called vector equations of the kinematic scheme.

a) Projection equations of the kinematic diagram:

Let O_1xy be a coordinate system associated with the price of the structure. The direction unit vectors of the O_1x and

O₁y axes are e_0, n_0 , respectively. The direction and direction of any vector e_x is determined by the angle φ_x that e_x makes with e_0 , the positive direction of the angle is clockwise (Figure 5).

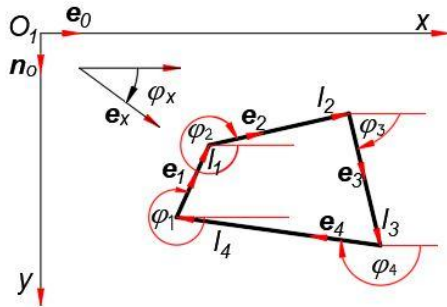


Figure 5: Kinematic diagram of 4-link structure

Scalar multiplication of the left side of the vector equation by e_0 and n_0 , respectively, we have:

$$\begin{cases} \sum_{i=1}^4 l_i e_i e_0 = 0 \\ \sum_{i=1}^4 l_i e_i n_0 = 0 \end{cases} \quad (9)$$

Because $e_i \cdot e_0 = \cos(e_i, e_0) = \cos\varphi_i$, $e_i \cdot n_0 = \cos(e_i, n_0) = \cos(\varphi_i - \pi/2) = \sin\varphi_i$, substituting into (9) we will get:

$$\begin{cases} \sum_{i=1}^4 l_i \cos\varphi_i = 0 \\ \sum_{i=1}^4 l_i \sin\varphi_i = 0 \end{cases} \quad (10)$$

b) Velocity equation

Consider the problem of calculating the velocity of a 4-link planar mechanism.

To solve the velocity calculation problem, first with the given assumptions we have determined the kinematic diagram polygon by solving the position calculation problem. After solving this calculation, the quantities l_i, e_i, φ_i , of the kinematic scheme polygon is completely determined. Taking the derivative with respect to t of the terms on the left of (8), we get:

$$\frac{d}{dt} \sum_{i=1}^3 l_i e_i = \sum_{i=1}^3 \left\{ \frac{dl_i}{dt} e_i + l_i \frac{de_i}{dt} \right\} = 0 \quad (11)$$

Set $\frac{dl_i}{dt} = \dot{l}_i$, and note $\frac{de_i}{dt} = \frac{d\varphi_i}{dt} n_i = \omega_i n_i$ that equation (11) can be rewritten as follows:

$$\sum_{i=1}^3 \{ \dot{l}_i e_i + \omega_i l_i n_i \} = 0 \quad (12)$$

To get the velocity projection equations, we take the dot product of the left side (12) in turn, we have:

$$\sum_{i=1}^3 \{ \dot{l}_i e_i + \omega_i l_i n_i \} \cdot e_0 = 0 \quad (13a)$$

$$\sum_{i=1}^3 \{ \dot{l}_i e_i + \omega_i l_i n_i \} \cdot n_0 = 0 \quad (13b)$$

Or:

$$\sum_{i=1}^3 \{ \dot{l}_i \cos\varphi_i + \omega_i l_i \sin\varphi_i \} = 0 \quad (14a)$$

$$\sum_{i=1}^3 \{ \dot{l}_i \sin\varphi_i + \omega_i l_i \cos\varphi_i \} = 0 \quad (14b)$$

These are the projection equations of the velocity of the planar 4-link mechanism in a coordinate system attached to the rack. After solving equations (14), we can deduce for each link, either the angular velocity and the linear velocity of a point belonging to it, or the long velocity of two points belonging to the link, which means we have solved the velocity calculation.

c) Acceleration equation

The content of the acceleration calculation is that given the kinematic size of the links, the position of the guide link, the angular acceleration of the guide link, and the acceleration of all links of the mechanism. The acceleration of a link is considered determined when we know:

- Or the angular acceleration of the link and the linear acceleration of any point on the link.
- Or the long acceleration of two points belonging to the link.

To solve the acceleration problem, we must first solve the position and velocity calculations, then the quantities $l_i, \varphi_i, \omega_i, \dot{l}_i$ are all known. Taking the derivative with respect to t of the terms on the left side of (12), we get:

$$\frac{d}{dt} \sum_{i=1}^3 (\omega_i \cdot l_i n_i + \dot{l}_i e_i) = 0 \quad (15)$$

Put $\varepsilon_i = \frac{d}{dt} \omega_i$; $\ddot{l}_i = \frac{d}{dt} \dot{l}_i$. On the other hand $\frac{d}{dt} n_i = -\omega_i e_i$. Therefore from the above equation we deduce:

$$\sum_{i=1}^3 (-\omega_i^2 l_i \mathbf{e}_i + \varepsilon_i l_i \mathbf{n}_i + 2\omega_i \dot{l}_i \mathbf{n}_i + \ddot{l}_i \mathbf{e}_i) = 0 \quad (16)$$

To obtain the projection equations of acceleration in an $O_i x_i y_i$ coordinate system associated with the price of the structure, we take the dot product of the left side of (16) with \mathbf{e}_o and \mathbf{n}_o , respectively, we get:

$$\begin{aligned} \sum_{i=1}^3 (-\omega_i^2 l_i \cos\varphi_i - \varepsilon_i l_i \sin\varphi_i - 2\omega_i \dot{l}_i \sin\varphi_i + \ddot{l}_i \cos\varphi_i) &= 0 \\ \sum_{i=1}^3 (-\omega_i^2 l_i \sin\varphi_i + \varepsilon_i l_i \cos\varphi_i + 2\omega_i \dot{l}_i \cos\varphi_i + \ddot{l}_i \sin\varphi_i) &= 0 \end{aligned} \quad (17)$$

After solving the system of equations (17) for each specific structure, we can determine the value of two quantities of the form. $\varepsilon_i, \ddot{l}_i$. With these two values and the value ε_i given in the hypothesis, we can deduce for each step is the long acceleration of two points or its angular acceleration and the long acceleration of a point belonging to it, that is, the acceleration calculation is then solved.

1.3 Dennaivit-Hartenberg-Craig matrix method

One of the commonly used methods to analyze the dynamics of multi-link systems (solid bodies) connected by joints is the Dennaivit-Hartenberg-Craig matrix method.

a) Determine the axes of the joint coordinate system

At the i th joint of a multi-body system, a joint coordinate system $(Oxyz)_i$ is built according to the following rule (Figure 6):

1. The z -axis $_i$ is selected along the i -th movable joint axis
2. The x_i axis is chosen along the common perpendicular line of the two axes z_i and z_{i+1} , direction from z_i to z_{i+1} .
3. The origin of coordinates O_i is the intersection of x_i and z_i . The y_i axis is chosen so that $(Oxyz)_i$ forms the forward reference frame.

b) Craig kinetic parameters

α_{i-1} : rotation angle around x_{i-1} axis so that z_{i-1} axis approaches z_i parallel to z_i .

a_{i-1} : translational displacement segment along the x_{i-1} axis so that the origin O_{i-1} approaches O_i (O_i is the intersection of the x_{i-1} and z_i axes).

θ_i : rotation angle around z_i axis so that x_{i-1} axis approaches x'_i (x'_i is parallel to x_i)

d_i : translational displacement segment along z_i axis so that x'_i axis approaches x_i axis.

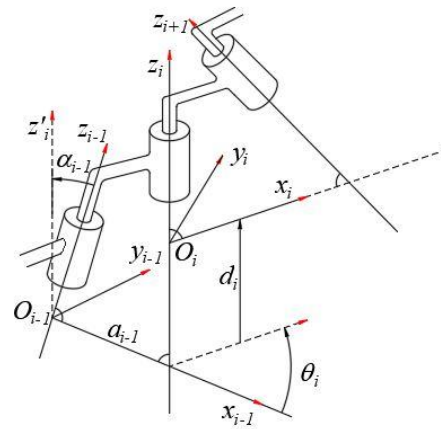


Figure 6: Coordinate system

c) Dennaivit-Hartenberg-Craig matrix

We can convert the joint coordinates $(Oxyz)_{i-1}$ to the joint coordinate system by 4 basic transformations: rotation around the x_{i-1} axis by an angle α_{i-1} , translation along the x_{i-1} axis by segment a_{i-1} , rotates around z_i axis by angle θ_i , translates along z_i axis by segment d_i . The \mathbf{K} transformation matrix is the product of 4 basic transformation matrices and has the following form:

$$\mathbf{K}_i = \begin{bmatrix} \cos\theta_i \sin\alpha_{i-1} - \sin\theta_i \cos\alpha_{i-1} & 0 & a_{i-1} \\ \sin\theta_i \sin\alpha_{i-1} + \cos\theta_i \cos\alpha_{i-1} & -\sin\alpha_{i-1} & -d_i \sin\alpha_{i-1} \\ \sin\theta_i \cos\alpha_{i-1} + \cos\theta_i \sin\alpha_{i-1} & \cos\alpha_{i-1} & d_i \cos\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

d) Location of the i^{th} link

Matrix \mathbf{K}_i shows the position of the i th link with respect to the $(i-1)^{\text{th}}$ link. For each step of the system, successively applying the transformations (18) we get: $\mathbf{C}_i = \mathbf{K}_1 \mathbf{K}_2 \dots \mathbf{K}_i$.

e) Velocity and acceleration of stages

When we know the global Dennaivit-Hartenberg-Craig matrix, we know the cosine matrix indicating the direction \mathbf{A}_i of the i^{th} link with respect to the fixed reference frame, from which we can calculate the angular velocity of the i^{th} solid object.

$$\tilde{\omega}_i^{(0)} = \dot{\mathbf{A}}_i \mathbf{A}_i^T \quad (19)$$

$$\tilde{\omega}_i^{(i)} = \mathbf{A}_i^T \dot{\mathbf{A}}_i \quad (20)$$

The velocity of the center of mass C_i of the i th solid object is determined according to equation (2.15).

$$\vec{v}_{C_i} = \vec{v}_{O_i} + \tilde{\omega}_i \times \vec{u}_{C_i} \quad (21)$$

When we know the angular velocity of the i^{th} solid object and the velocity of its center of mass, by calculating the derivative we can easily determine the angular acceleration and center-of-mass acceleration of the i^{th} solid object.

Graph and vector diagram methods [1], [2] are mainly applied to flat structures. Currently, the popular method in dynamic analysis is the analytical method [3], [4]. However, the analytical method also encounters difficulties when establishing equations associated with spatial structures complicated. Therefore, this study proposed an approximate method, called “the perpendicular projection method”. This is a combination of geometric and analytical methods, used to analyze spatial structure kinematic, with the purpose of helping to solve the problem of spatial analysis. Kinematic calculus can be solved in a simpler and easier way.

II. KINEMATIC ANALYSIS OF SPATIAL STRUCTURE USING THE PERPENDICULAR PROJECTION METHOD

Each spatial structure is a mechanical system subject to holding and stopping holonomic linkage, its constraint conditions are of the form:

$$\sum l_i = 0 \tag{22}$$

We used the sufficiently generalized coordinates $q_1, q_2, q_3, \dots, q_f$ to determine the position of the structure f degrees of freedom. In addition, we added extra generalized coordinates to easily establish analytical expressions to determine the position of the structure. Let $z_1, z_2, z_3, \dots, z_r$ be the dependent generalized coordinates. The connecting equations have the form:

$$f_i(q_1, q_2, \dots, q_f, z_1, z_2, \dots, z_r) = 0 \quad (i = 1 \div r) \tag{23}$$

Where:

$$z_k = z_k(q_1, q_2, \dots, q_r) \quad (k = 1 \div r) \tag{24}$$

Solving the system of nonlinear algebraic equations (24) to determine the functions z_i and its first and second derivatives using the iterative method is the central task of the spatial structural kinematics analysis problem.

a) Oxyz fixed coordinate system

This method is based on the properties of perpendicular projection to establish a fixed coordinate system with the purpose of reducing the number of redundant generalized coordinates. The principles are as follows: If connecting a rotating rack around a fixed axis, choose a rotating surface

parallel to the rotating axis (Figure 7). If the price connection is translational, or both rotating and translating, choose one of the two x or y axes parallel to the translation direction (Figure 8).

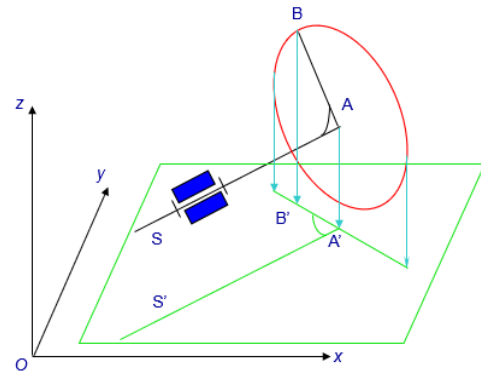


Figure 7: Connection of rotating rack around fixed axis

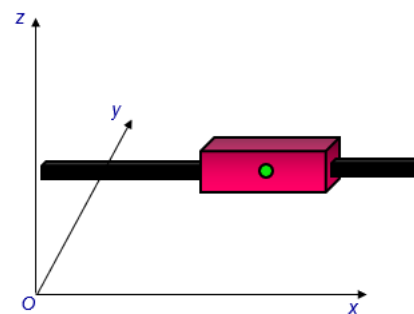


Figure 8: Connection of translational rack

b) Residual generalized coordinates

For each link, it is necessary to determine the angle between the projection of a coordinate axis attached to the link with the Ox (or Oy) axis, and the angle of the coordinate axis attached to that link with the Oz axis (Figure 9). According to Figure 9, calling the length of AB of l , it is easy to determine the projection of segment AB onto the x, y and z axes as $l \sin \beta \cos \alpha, l \sin \beta \cos \alpha$ and $l \cos \beta$, respectively. If there is a translational joint, it is necessary to choose additional coordinates such as long displacement (Figure 10).

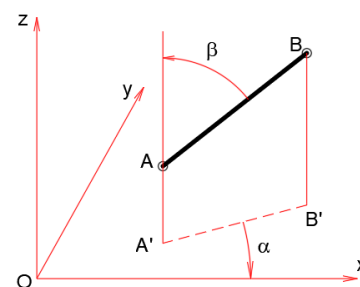


Figure 9: Residual generalized coordinates

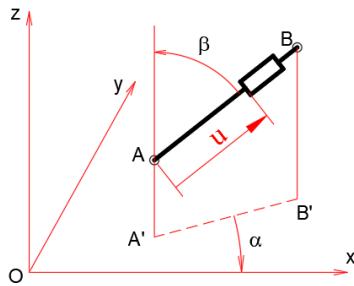


Figure 10: Generalized coordinates when there is a translational joint

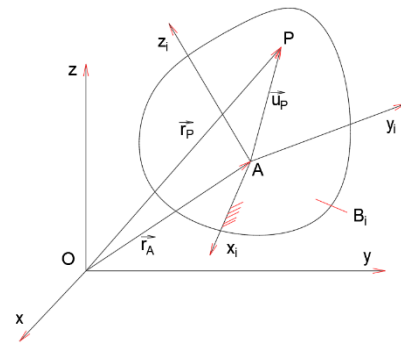


Figure 11: Determine the location of point P

c) Angular velocity, angular acceleration of the stages

The angular velocity of a link is determined according to the matrix formula (25). Where A is the cosine matrix indicating the direction of the i^{th} link; $\tilde{\omega}_i = [\omega_x, \omega_y, \omega_z]^T$; $\omega_x, \omega_y, \omega_z$ are the projections of the angular velocity vector onto the coordinate axes, respectively.

$$\tilde{\omega}_i = \dot{A}_i A_i^T \quad (25)$$

The derivative of angular velocity with respect to time will obtain the angular acceleration of the link.

d) Position, velocity and acceleration of a point on a link

The position of a point P on the i^{th} link (Figure 11) can be determined according to the matrix expression (26):

$$r_p = r_A + u_p \quad (26)$$

In which r_p, r_A are column matrices of x, y, z coordinates of point P and point A, respectively, u_p is column matrix of vector coordinates \vec{u}_p .

On the other hand, $u_p = A_i \cdot u^{(i)}$, where A_i is the cosine matrix indicating the direction of link i, $u^{(i)}$ is the coordinate column matrix of point B for the i^{th} coordinate system. Thus, expression (26) can be written in the form of expression (27).

$$r_p = r_A + A_i \cdot u^{(i)} \quad (27)$$

The velocity of a point in the i^{th} phase is calculated according to formula (28).

$$v_p = v_A + \dot{A} u_p^i \quad (28)$$

In which v_p is the velocity of point P, v_A is the velocity of point A in matrix form, u_p^i which is a column matrix consisting of 3 components of the location vector of point P in the i^{th} coordinate system. The derivative of velocity with respect to time will obtain the acceleration of the point. The steps taken when analyzing kinematics using the perpendicular projection method are shown in Figure 12.

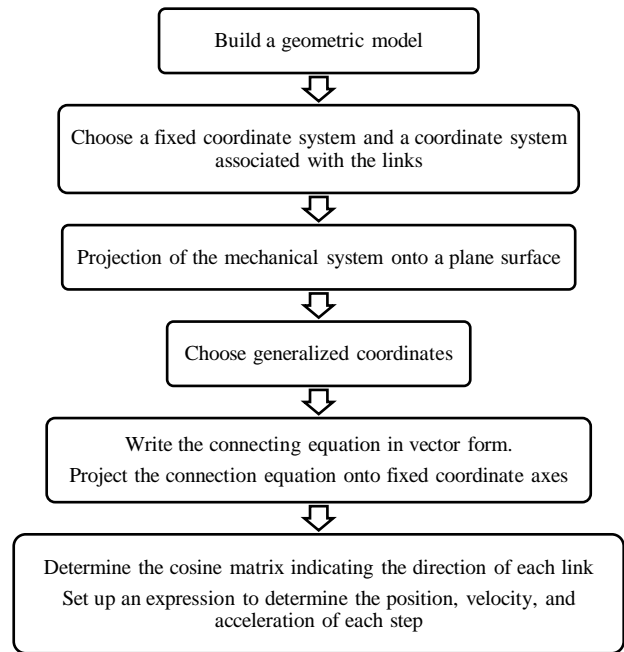


Figure 12: Diagram of the method steps

III. KINEMATIC ANALYSIS OF SPATIAL STRUCTURE WITH 1 ROTATING JOINT, 1 SPHERICAL JOINT, 1 TRANSLATIONAL JOINT AND 1 CYLINDRICAL JOINT (RSPC)

This section presents the application of the perpendicular projection method to solve a kinematic analysis of spatial structure with 1 rotating joint, 1 spherical joint, 1 translational joint and 1 cylindrical joint (RSPC)

a) Geometric model

Figures 13 and Figures 14 depict a 4-step RSPC spatial structure. The first link is SAB in which AB is perpendicular to SA, length $AB=l_1$. Sew SAB rotates around SA axis. The second step is BC. Stage BC moves relative to stage CDE. The third link is CDE in which CD is perpendicular to DE, length $CD=l_2$, $DF=a$, $CF=b$. The CDE scene just filmed around DE just translates along DE. The joint at B is a ball joint, the joint

at C is a translation joint. C_1, C_2, C_3 are the focus of the first, second and third stages, respectively.

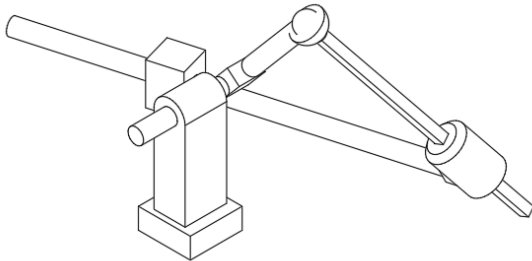


Figure 13: RSPC 4-stage structural model

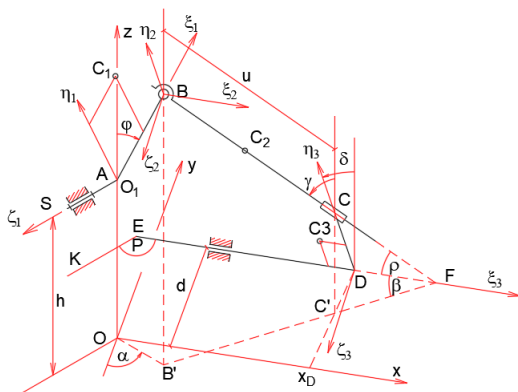


Figure 14: RSPC 4-step structure puzzle

b) Oxyz fixed coordinate system and axis systems attached to the links

Through E, draw line EK parallel to SA. KED forms plane P. Project point A perpendicular to plane P to get point O. Through O, draw Ox parallel to DE, Oy perpendicular to Ox and Oz passes through OA. Select the axis systems attached to the links as shown in Figure 3.8, in which axis system $A\xi_1\eta_1\zeta_1$ is attached to link SAB, axis system $B\xi_2\eta_2\zeta_2$ is attached to link BC, axis system $D\xi_3\eta_3\zeta_3$ is attached to link CDE. Link BC moves relative to link CDE, so the axis system $B\xi_2\eta_2\zeta_2$ is taken parallel to the axis system $D\xi_3\eta_3\zeta_3$.

c) Projecting the mechanical system onto the xOy plane

The projections of point O, point B, point D and point E onto the xOy plane are points O, B', C', D and E, respectively.

d) Choose generalized coordinates

Choose the sufficiently generalized coordinates as φ , the residual generalized coordinates are $\beta = \beta(\varphi)$, $\gamma = \gamma(\varphi)$, $\delta = \delta(\varphi)$, $u = u(\varphi)$, $x_D = x_D(\varphi)$.

e) The equation linking vector form and projection form

From Figure 3.8, we have the motion constraint equation of the vector structure:

$$\vec{OA} + \vec{AB} + \vec{BC} + \vec{CD} + \vec{DO} = 0 \quad (29)$$

and:

$$\vec{DC} + \vec{CF} + \vec{FD} = 0 \quad (30)$$

Projecting equations (3.7) and (3.8) onto the x, y, z coordinate axes, we have the following equations:

$$l_1 \sin\varphi \sin\alpha + u \sin\gamma \cos\beta - x_D = 0 \quad (31)$$

$$-l_1 \sin\varphi \cos\alpha + u \sin\gamma \sin\beta + l_2 \sin\delta - d = 0 \quad (32)$$

$$h + l_1 \cos\varphi - u \cos\gamma - l_2 \cos\delta = 0 \quad (33)$$

$$b \cos\rho - a = 0 \quad (34)$$

$$-l_2 \sin\delta + b \sin\gamma \sin\beta = 0 \quad (35)$$

$$l_2 \cos\delta - b \cos\gamma = 0 \quad (36)$$

Equation (34) contains constants that always hold true for right triangle CDF. Equations (31), (32), (33), (35), (36) are the geometric connection equations of the structure.

$$\begin{cases} l_1 \sin\varphi \sin\alpha + u \sin\gamma \cos\beta - x_D = 0 \\ -l_1 \sin\varphi \cos\alpha + u \sin\gamma \sin\beta + l_2 \sin\delta - d = 0 \\ h + l_1 \cos\varphi - u \cos\gamma - l_2 \cos\delta = 0 \\ -l_2 \sin\delta + b \sin\gamma \sin\beta = 0 \\ l_2 \cos\delta - b \cos\gamma = 0 \end{cases} \quad (37)$$

f) Determine the residual generalized coordinates and their velocities and accelerations

Derivative (37) according to φ get:

$$\begin{cases} l_1 \cos\varphi \sin\alpha + u' \sin\gamma \cos\beta + u\gamma' \cos\gamma \cos\beta - u\beta' \sin\gamma \sin\beta - x'_D \\ -l_1 \cos\varphi \cos\alpha + u' \sin\gamma \sin\beta + u\gamma' \cos\gamma \sin\beta + u\beta' \sin\gamma \cos\beta + l_2 \delta' \sin\delta \\ -l_1 \sin\varphi - u' \cos\gamma + u\gamma' \sin\gamma + l_2 \delta' \sin\delta \\ -l_2 \delta' \cos\delta + b\gamma' \cos\gamma \sin\beta + b\beta' \sin\gamma \cos\beta \\ -l_2 \delta' \sin\delta + b\gamma' \sin\gamma \end{cases} \quad (38)$$

or write as:

$$J_z z' = d_1 \quad (39)$$

Where:

$$J_z = \begin{bmatrix} u \cos\gamma \cos\beta & -u \sin\gamma \sin\beta & 0 & \sin\gamma \cos\beta & -1 \\ u \cos\gamma \sin\beta & u \sin\gamma \cos\beta & l_2 \cos\delta & \sin\gamma \sin\beta & 0 \\ u \sin\gamma & 0 & l_2 \sin\delta & -\cos\gamma & 0 \\ b \cos\gamma \sin\beta & b \sin\gamma \cos\beta & -l_2 \cos\delta & 0 & 0 \\ b \sin\gamma \sin\beta & 0 & -l_2 \sin\delta & 0 & 0 \end{bmatrix}$$

$$z' = \begin{bmatrix} \gamma' \\ \beta' \\ \delta' \\ u' \\ x''_D \end{bmatrix}, d_1 = \begin{bmatrix} l_1 \cos\varphi \sin\alpha \\ l_1 \cos\varphi \cos\alpha \\ l_1 \sin\varphi \\ 0 \\ 0 \end{bmatrix}$$

Solving equation (39), we find z' , then determine \dot{z} according to equation $\dot{z} = z' \dot{\varphi}$.

Derivative (39) according to φ has:

$$J_z z'' = d_2 \tag{40}$$

where: $z'' = \begin{bmatrix} \gamma'' \\ \beta'' \\ \delta'' \\ u'' \\ x''_D \end{bmatrix}$

$$d_2 = \begin{bmatrix} l_1 \sin\varphi \sin\alpha - 2u' \gamma' \cos\varphi \cos\beta + 2u' \beta' \sin\varphi \sin\beta + u\gamma'^2 \sin\varphi \cos\beta + 2u\gamma' \beta' \cos\varphi \sin\beta + u\beta'^2 \sin\varphi \cos\beta \\ -l_1 \cos\varphi \cos\alpha - 2u' \gamma' \cos\varphi \sin\beta - 2u' \beta' \sin\varphi \cos\beta + u\gamma'^2 \sin\varphi \sin\beta - 2u\gamma' \beta' \cos\varphi \cos\beta + u\beta'^2 \sin\varphi \sin\beta + l_2 \delta'^2 \sin\delta \\ l_1 \cos\varphi - 2u' \gamma' \sin\gamma - u\gamma'^2 \cos\gamma - l_2 \delta'^2 \cos\delta \\ -l_2 \delta'^2 \sin\delta + b\gamma'^2 \sin\gamma \sin\beta - 2b\gamma' \beta' \cos\varphi \cos\beta + b\beta'^2 \sin\gamma \sin\beta \\ l_2 \delta'^2 \cos\delta - b\gamma'^2 \cos\beta \end{bmatrix}$$

Solving equation (40) we get z'' , which can then be determined \ddot{z} according to the equation: $\ddot{z} = z'' \dot{\varphi}^2 + z' \ddot{\varphi}$.

The functions $\gamma(\varphi), \beta(\varphi), \delta(\varphi)$ are approximately defined according to the Taylor standard as follows:

$$\gamma(\varphi_0 + \Delta\varphi) = \gamma(\varphi_0) + \gamma'(\varphi_0)\Delta\varphi + \frac{1}{2}\gamma''(\varphi_0)\Delta\varphi^2 + \dots$$

$$\beta(\varphi_0 + \Delta\varphi) = \beta(\varphi_0) + \beta'(\varphi_0)\Delta\varphi + \frac{1}{2}\beta''(\varphi_0)\Delta\varphi^2 + \dots$$

$$\delta(\varphi_0 + \Delta\varphi) = \delta(\varphi_0) + \delta'(\varphi_0)\Delta\varphi + \frac{1}{2}\delta''(\varphi_0)\Delta\varphi^2 + \dots$$

Then determine more accurately using the Newton iteration method.

g) Center of mass position of the links

According to Figure 14, the direction cosine matrix of the SAB link is determined by 3 consecutive rotations:

- Rotate around the oz axis with angle $(90^\circ - \alpha)$, in the negative direction.

$$A_{11} = \begin{bmatrix} \sin\alpha & \cos\alpha & 0 \\ -\cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Rotate around oy axis with angle $(90^\circ - \varphi)$, in the negative direction.

$$A_{12} = \begin{bmatrix} \sin\varphi & 0 & -\cos\varphi \\ 0 & 1 & 0 \\ \cos\varphi & 0 & \sin\varphi \end{bmatrix}$$

- Rotate around the ox axis with an angle of 90° , in the positive direction.

$$A_{13} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

As a result, we can calculate the direction cosine matrix of the SAB link as follows:

$$A_1 = A_{11} \cdot A_{12} \cdot A_{13} = \begin{bmatrix} \sin\alpha \sin\varphi & -\sin\alpha \cos\varphi & -\cos\alpha \\ -\cos\alpha \sin\varphi & \cos\alpha \cos\varphi & -\sin\alpha \\ \cos\varphi & \sin\varphi & 0 \end{bmatrix} \tag{41}$$

The direction cosine matrix of link CDE (and of link BC) is determined through rotation around Ox at an angle of $180^\circ + \delta$, negative direction:

$$A_3 = A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\delta & \sin\delta \\ 0 & -\sin\delta & -\cos\delta \end{bmatrix} \tag{42}$$

Let C_1 be the center of mass point of the SAB link and assume that in the $A_1 \xi_1 \eta_1 \zeta_1$ coordinate system, the coordinates of C_1 are $(\xi_{11}, \eta_{11}, 0)$. The equation determining C_1 in a fixed coordinate system can be written in matrix form (43) or projection form (44).

$$r_{C1} = r_A + A_1 \cdot u \tag{43}$$

$$\begin{aligned} x_{C1} &= \xi_{11} \sin\varphi \sin\alpha - \eta_{11} \cos\varphi \sin\alpha \\ y_{C1} &= -\xi_{11} \sin\varphi \cos\alpha + \eta_{11} \cos\varphi \cos\alpha \\ z_{C1} &= h + \xi_{11} \cos\varphi + \eta_{11} \sin\varphi \end{aligned} \tag{44}$$

Let C_2 be the center of mass point of link BC and assume, in the $B_2 \xi_2 \eta_2 \zeta_2$ coordinate system, the coordinates of C_2 are $(\xi_{22}, \eta_{22}, 0)$. The equation for determining C_2 in a fixed coordinate system can be written in matrix form (45) or projection form (46).

$$r_{C2} = r_B + A_2 \cdot u \tag{45}$$

$$\begin{cases} x_{C2} = l_1 \sin\varphi \sin\alpha + \xi_{22} \\ y_{C2} = -l_1 \sin\varphi \cos\alpha - \eta_{22} \cos\delta \\ z_{C2} = h + l_1 \cos\varphi - \eta_{22} \sin\delta \end{cases} \tag{46}$$

Let C_3 be the center of mass point of link CDE and assume, in the $D_3 \xi_3 \eta_3 \zeta_3$ coordinate system, the coordinates of C_3 are $(\xi_{33}, \eta_{33}, 0)$. The equation for determining C_3 in a fixed coordinate system can be written in matrix form (47) or projection form (48).

$$r_{C3} = r_D + A_3 \cdot u \tag{47}$$

$$\begin{cases} x_{C3} = & l_1 \sin\varphi \sin\alpha + u \sin\gamma \cos\beta + \xi_{33} \\ y_{C3} = & -l_1 \sin\varphi \cos\alpha + u \sin\gamma \sin\beta + l_3 \sin\delta - \eta_{33} \cos\delta \\ z_{C3} = & h + l_1 \cos\varphi - u \cos\gamma - \eta_{33} \sin\delta \end{cases} \quad (48)$$

h) Velocity and acceleration of the center of mass of the links

Differentiating the expressions (44), (46), (48) over time, we have equations determining the velocities of points C_1 , C_2 , C_3 . After that, differentiating the equations determining the velocities of points C_1 , C_2 , C_3 over time, we will obtain equations determining the acceleration of those points.

i) Angular velocity and angular acceleration of the links

Suppose the SAB guide rotates with angular velocity $\omega_1 = \dot{\varphi}$ and angular acceleration $\varepsilon_1 = \ddot{\varphi}$. We can determine the angular velocity according to the following equation:

$$\tilde{\omega}_1 = \begin{bmatrix} 0 & -\omega_{1z} & \omega_{1y} \\ \omega_{1z} & 0 & -\omega_{1x} \\ -\omega_{1y} & \omega_{1x} & 0 \end{bmatrix} = A_1 A_1^T = \begin{bmatrix} 0 & 0 & \dot{\varphi} \sin\alpha \\ 0 & 0 & -\dot{\varphi} \cos\alpha \\ -\dot{\varphi} \sin\alpha & \dot{\varphi} \cos\alpha & 0 \end{bmatrix} \quad (49)$$

From (49), we have:

$$\omega_1 = \begin{bmatrix} \omega_{1x} \\ \omega_{1y} \\ \omega_{1z} \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{bmatrix} \dot{\varphi}, \varepsilon_1 = \begin{bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{bmatrix} \ddot{\varphi} \quad (50)$$

Calculating similarly for stages BC and CDE, we have:

$$\tilde{\omega}_3 = \begin{bmatrix} 0 & -\omega_{3z} & \omega_{3y} \\ \omega_{3z} & 0 & -\omega_{3x} \\ -\omega_{3y} & \omega_{3x} & 0 \end{bmatrix} = A_3 A_3^T \quad (51)$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\dot{\delta} \\ 0 & \dot{\delta} & 0 \end{bmatrix}$$

$$\omega_3 = \begin{bmatrix} \omega_{3x} \\ \omega_{3y} \\ \omega_{3z} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \dot{\delta}, \varepsilon_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \ddot{\delta} \quad (52)$$

IV. DISCUSSIONS AND CONCLUSION

This article introduced some methods of structural kinematic analysis. Each of these methods has its own advantages and disadvantages and its own fields of application. However, they have the characteristic that they are difficult to apply to complex spatial structures. In this article, the author proposed a method that combines the advantages of the above methods, using both vectors, projections, and the matrix method, called the method of perpendicular projections to differentiate structural kinematic. By optimally choosing the fixed coordinate system, residual coordinates can be reduced while still easily establishing the equation of motion of the mechanism. Besides that, by choosing the residual coordinates of the method, projecting the vector connection equation onto the coordinate axes also

becomes very simple. The article applied the perpendicular projection method to analyze the dynamics of a specific spatial structure. By choosing the optimal fixed coordinate system, the angle α , x_D coordinate, y_D coordinate became fixed constants, which means reducing the 3 necessary residual coordinates. This method is simple so it can be used by technical workers, engineers, and researchers. In addition, using this method it is easy to program and simulate machine movements.

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